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The Effect of Perspective Geometry on Judged Direction in Spatial Information Instruments

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As part of a study of spatial information transfer, eight subjects judged the directions of displayed targets relative to a fixed reference position in the center of each of 640 perspective images. The stimulus images subtended 18 deg of the observer's visual field, while the images were constructed with geometric fields of view ranging from 30 to 120 deg. Target elevation is consistently overestimated, especially in "telephoto" images. Azimuth error varies sinusoidally with the azimuth direction of the target, alternating between clockwise and counterclockwise errors from one direction quadrant to the next. The direction of this azimuth error gradually reverses in each quadrant as the perspective is varied between "telephoto" and "wide angle" views, so that clockwise azimuth error becomes counterclockwise error, and vice versa. The amplitude of the sinusoidal azimuth error is least in the images with a 60-deg field of view. We propose a geometrical model of an interpretive behavior associated with viewing perspective displays in which the sinusoidal pattern of azimuth errors is induced by the difference between the 3D stimulus and its 2D projection, and by the consequences of the geometric differences between the station point and the observer's actual eye position.

INTRODUCTION

Use of pictures as spatial information instruments has been of particular interest in aerospace applications (Getty, 1982; Jauer and Quinn, 1982; Jones, Schrader, and Marshall, 1950; Roscoe, Corl, and Jensen, 1981; Warner, 1979). Primary tasks in aerospace involve maneuvering through three-dimensional space amid a variety of fixed or moving objects. These include physical objects like aircraft, missiles, mountains, or weather systems, and virtual objects like traffic control or threat zones. Assistance in monitoring the spatial relationships among objects of interest can best be provided by instruments that match both the spatial dimensions of the tasks for which they are used and human perceptual capabilities.

Current spatial displays, such as the navigation display on the Boeing 767, typically map two dimensions of the airspace onto the two dimensions of the flat display panel and encode or ignore the collapsed vertical dimension. While this may be adequate for lateral navigation information, it is not well suited to vertical navigation use. Presenting traffic information on such a planview display has resulted in horizontally biased avoidance maneuvers (Ellis, McGreevy, and Hitchcock, 1984; Smith, Ellis, and Lee, 1984).

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A recent experimental format (Figure 1) uses synthesized perspective views to present the horizontal and vertical traffic situation in an integrated format (McGreevy, 1983a, 1983b; McGreevy and Ellis, in press). Horizontal maneuver biases found in planview displays are reduced considerably with this perspective format (Ellis et al., 1984).

Simple spatial appearance does not ensure effective spatial information transfer between the system and the user. When three-dimensional information is projected onto a two-dimensional screen, the original information must be mentally reconstructed by the user. No matter how accurate the database, the user may introduce distortions in the act of interpretation of the projection. Complicating the design of perspective displays is the fact that the appearance of an image varies dramatically as a function of the perspective parameter values used to generate it.

The purpose of the following experiment was to determine whether the differences in

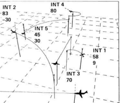

Figure 1. *A perspective display of air traffic for the cockpit that has been used in our format studies.*

appearance that are due to these perspective parameters result in differences of spatial interpretation (McGreevy and Ellis, 1984). Specifically, we chose to test the hypothesis that direction judgment error in perspective displays varies as a function of the geometric parameters of perspective.

METHOD

Stimulus Description

The format of the stimulus images was abstracted from our air traffic display format (Figure 1) so that the task would require judgments similar to those needed in a practical spatial information instrument. The perspective images used in the experiment were generated on a high-resolution cathode-ray tube. Each image consisted of two cubes, a horizontal grid, and metric lines connecting the cubes to the grid (Figure 2). The two cubes were the same size in three-dimensional space, although perspective effects could cause their two-dimensional projections to differ in size on the screen. The reference cube was always at the center of the screen above the center of the grid, whereas the target cube was positioned at a constant radial distance from the reference cube in one of 40 possible directions. The perspective geometry of the stimulus images was varied in a manner analogous to that achieved by using four different settings of a camera's zoom lens at four different distances from the reference cube, for a total of 16 different perspectives. The high-contrast images were drawn with white lines on a black background of less than 3 cd/m². The bright cubes and their metric lines (approximately 150 cd/m²) stood out clearly against the grid, whose luminance varied with scene depth from approximately 50 cd/m² in the foreground to 10 cd/m² in the background.

The subjects were asked to adjust the pointers in two round dials to indicate the

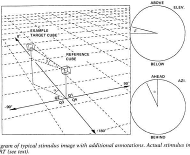

Figure 2. *Diagram of typical stimulus image with additional annotations. Actual stimulus images were generated on a CRT (see text).*

judged elevation and azimuth angles of the target cube relative to the reference cube, so as to indicate the direction of the target relative to the reference cube. Target azimuth was defined as the angle between the zero degrees azimuth direction (the "heading" of the reference cube) and the horizontal direction to the target cube (see Figure 2). Target elevation was defined as the angle between the altitude plane of the reference cube and the vertical direction to the target cube.

The cubes, their radial separation, and their distance above the grid were scaled so as to hold the screen extent of the directional stimuli constant while varying the perspective. This allowed us to measure the effect of perspective "distortion" separately from the *global* scaling effects of perspective. Only the grid was allowed to be globally scaled by the perspective. All local scaling effects of perspective were preserved. Thus, the cubes were scaled so that, regardless of perspective, the reference cube image size remained constant. This scaling still allowed the target cube to vary in size relative to the reference cube as a function of perspective, leaving the depth cue of relative size intact. The radial distance between the target and reference cubes was scaled so that the sphere of loci of the target cube within each perspective was just the right size to nearly reach the edges of the image and still allow the target cube to remain within the displayed volume. The screen distance between the reference cube and the grid was scaled so that, regardless of perspective, the grid appeared to remain at a constant distance below the reference.

Other elements were added to the display to minimize confounding cues or to serve as references. The cubes were slowly tumbled, so that the orientation of the cubes in the projected images would not favor one perspective or direction over another. The grid lines perpendicular to the heading were moved in a direction opposite to the heading; this was done to make the display similar to our previously defined cockpit air traffic display and to help defeat measuring strategies. A crossshaped symbol intersected the line connecting the target cube with the grid, indicating the altitude of the reference cube. This was provided so that subjects judging elevation to the target would not be required to also determine the reference cube's altitude at the target's position. When targets went below the grid, a little grid symbol marked the spot where the metric line passed through the grid.

The grid and cube scene was viewed from an azimuth of −158 deg relative to the reference cube "heading" of zero deg azimuth, and from an elevation of 22 deg above the reference cube altitude above the grid. The view was therefore directed down toward the grid, along the 22-deg azimuth line. These viewing parameters were chosen because they were similar to those that seemed to be useful in the traffic situation display used in previous studies.

Subjects and Procedure

The eight subjects included five commercial airline pilots, one general aviation pilot, and two nonpilots. One of the nonpilots was female. The commercial pilots were randomly selected, and the other three subjects were laboratory personnel. To familiarize the subjects with the display format and the task, each was asked to read a booklet that explained the task and contained figures that graphically described the stimulus display contents and relationships of interest.

After reading the booklet, the subject was shown a few representative stimulus images and was asked questions, and corrected if necessary, to ensure that the information was understood. The subject was then allowed to try the task a few times, providing sufficient feedback for investigators to verify that the task was understood. To ensure that the indicated direction was identical to the intended indication, a temporary third cube, the training cube, which showed the judged position, was added to the scene for comparison with the intended indication of direction.

Subjects were instructed that accuracy was more important than speed, but were advised to respond quickly so that the 640 trials would not take an excessive amount of time. They were instructed to take a short break at any time rather than to become sloppy or hurried. Subjects were checked at intervals to make sure that they were not getting tired, and they were given regular breaks. The subjects were told to respond with their immediate impression and to avoid any measuring strategies. We also assured the pilot subjects that the experiment was not a test of their skill level and had no reflection on the adequacy of their piloting performance, so that they need not worry, as some do, about passing the "test."

Apparatus

During the experiment, the subject was seated at a table that was in front of the 53.3-cm display monitor. The distance between the subject's eyes and the monitor was approximately 61 cm. For indicating the direction judgments, the subject had a digitizer pad and stylus, which were connected by software to a pair of circular dials drawn on the screen. One dial indicated target azimuth direction, and the other was for target elevation direction. The subject used a momentary action switch on a switch panel to indicate readiness for the next trial.

The stimulus image was 19.1 cm square, and the two response dials, each 7.6 cm in diameter, were drawn to one side of the image (Figure 2). The stimulus generation and data acquisition program was written in OMSI Pascal by one of the authors (McGreevy) and ran on a Digital Equipment Corporation (DEC) PDP-11/70, under the DEC RSX-11M real-time operating system. The computer graphics images were generated on an Evans and Sutherland Picture System 2

(E&S PS2), a dynamic, calligraphic display processor. The E&S display monitor has a resolution of 4096 × 4096, and can update thousands of vectors at a rate sufficient for smoothly dynamic imagery.

Stimulus Geometry

By using computer graphics techniques, the stimulus geometry in this experiment is thoroughly quantified, so that the three-dimensional coordinates of elements in the displayed scenes and the projected two-dimensional coordinates are precisely known. Furthermore, the exact nature of the viewing transformations is completely quantified, so that every rotation, translation, scaling, and projection parameter is available.

This quantitative description allows creation of stimuli to precise specifications and is essential for use in modeling the effects of different viewing geometries. The description of the stimulus geometry is quite complex, however, and requires a great deal of spatial visualization and the use of a sometimes confusing and not well-standardized vocabulary. For more information, the interested reader is urged to refer to a computer graphics text (e.g., Foley and Van Dam, 1982, or Newman and Sproull, 1979) or to the author's description of a similar display (McGreevy, 1983; McGreevy and Ellis, in press).

Figure 3 illustrates the stimulus geometry. The inset shows a scene similar to those used in the experiment. Such an image is created as the result of projecting each point in the three-dimensional scene to the *station point*, also known as the *center of projection*, and intersecting the *projectors* with a plane called the *picture plane*. The station point is the point through which all imaged projectors pass, just as a pinhole lens is the point through which all imaged light rays pass. It is a convention in computer graphics to place the picture plane between the scene and the

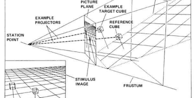

Figure 3. *Example of stimulus perspective geometry, showing relationships between the three-dimensional stimulus and its two-dimensional projection.*

station point, where it intersects the projectors; this has the advantage of presenting an upright image.

The *field of view angle* is defined as the visual angle of the image as seen from the station point, and was varied as part of the perspective conditions of the experiment. The vertical field of view will differ from the horizontal if the image is not square, so for simplicity a square image was used. Each edge of the image, together with the station point, defines an *edge clipping plane*, so-called because scene objects that pass through such a plane are clipped into displayed and undisplayed segments. Projectors from visible parts of the object project to the station point *within* the boundaries of the image, whereas undisplayed parts are those whose projectors reach the station point by passing *outside* the boundaries of the image. Everything in an edge clipping plane projects onto the edge of the image.

The part of the picture plane bounded by the edges of the image is usually referred to as a *window*. By contrast, the part of the monitor screen onto which the image is drawn is called a *viewport*. This distinction is important because the size, shape, and location of the window, which exists in three-dimensional display space, determines what information is visible at the station point, whereas the viewport is the region of the display screen onto which this information is mapped. Notice that the proximity of a scene element to a side clipping plane in three dimensions determines the proximity of the image of that element to the edge of the window or viewport.

An optional clipping plane was defined that corresponds to the picture plane. This plane is usually called the *hither clipping plane*. Nothing between the station point and the hither plane is displayed. This is compatible with the actual viewing situation where the spatial scene appears to be behind the glass face of the CRT, and no objects appear on the near side of the glass.

The shape of the visible volume of display space carved out by the four edge clipping planes and the hither plane is a *frustum*, that is, a truncated pyramid, whose bottom is either bounded by a *yon clipping plane* or is at infinity. The apex of the pyramid is the station point. Selecting the angle between opposite edge clipping planes at the apex determines the field-of-view angle and the particular shape of the frustum of visible space. The station point and field-of-view angle used to create an image are independent of the actual eye position of an observer viewing the image.

The position of the reference cube is the *reference point*, which is directly above the center of the grid in stimulus images used in this experiment. The projector from this point passes through the center of the image to the station point and is orthogonal to the picture plane. The distance between the reference point and the station point may vary, which is like changing the distance between a camera and its subject. This distance was varied as part of the perspective conditions of the experiment.

Experiment Design

The experiment was a fully crossed, repeated measures design. Each of the eight subjects was shown 640 stimulus images, which were obtained by crossing 16 perspective conditions with 40 direction conditions. The 16 perspectives were obtained by crossing four fields of view with four distances between the reference cube and the station point position in the display space.

The four distances were 1000, 7333, 13 667, and 20 000 display space units. These arbitrary units relate the four distances to the size of the grid, which was composed of an array of 24 × 24 grid squares, each of which was 2730 units square. Due to the scaling, which kept the cube sizes and separation distances consistent across different fields of view, varying the distance between the station point and the reference cube amounted to varying the global scaling of the grid.

When we refer to *grid scaling,* we mean the visual effect on the grid of putting the station point at various geometric distances from the grid, which is the computer graphics equivalent of viewing the grid from various distances. A more distant station point causes the grid to appear more dense in the image, and a more proximal view causes the grid to appear less dense. Changing grid scaling in this way does not have any effect on the convergence of the lines in the grid.

Changing the field-of-view angle changes the convergence of the grid, as well as its density in the image, so we do not refer to this effect as grid scaling. A wider field of view takes in more of the grid at any given distance and creates an image with a denser, more strongly converging grid. Conversely, a more narrow field of view takes in less of the grid at any given distance and creates an image of a less dense, weakly converging grid. For example, the grids in the images in Figure 4 do not differ in scaling because they are all constructed with the same station point to reference point distance (7333), and the differences between these grids are due only to the different fields of view.

The four field of view conditions were 30, 60, 90, 120 deg, where 30 deg is similar in effect to a telephoto lens and 120 deg is approaching a fish-eye lens effect. Since the stimulus image was 19.1 cm square, the station points were positioned at four distinct distances from the display screen: 35.6, 16.5, 9.7, and 5.6 cm, respectively.

The station point and field-of-view angle used to create an image are independent of the actual eye position of an observer viewing the image. Each subject's actual eye position was approximately 61 cm from the screen, and although no head restraint was used, the distance was maintained reasonably well. The stimulus image subtended a visual angle of approximately 18 deg. Thus, at the four different fields of view the station point was at four different distances from the subject's fixed eyepoint.

The 40 direction conditions were obtained by crossing 8 azimuths with 5 elevations. The sphere of all possible target directions was divided into 40 direction regions by crossing 8 azimuth regions with 5 elevation regions. Boundaries of the azimuth regions were: 0, ± 45, ± 90, ± 135, and ± 180 deg. Boundaries of the elevation regions were: ± 15, ± 45, and ± 75 deg. No sampling was done at elevations beyond ± 75 deg. Target cubes were presented in all 40 direction regions for each of the 16 perspective conditions. This ensured that any judgment performance differences among the perspectives would be based on a wide variety of direction judgments. For purposes of the original hypothesis, the 40 direction judgments were considered replications within a perspective. Thus, we had eight subjects making 40 judgments in each of 16 perspective conditions.

Figure 4 shows four example stimulus images, which differ in field of view. The distance between the station point and the reference point is 7333 display space units in these images. The center cube is the reference cube. The cube on the left is a training cube whose direction is identical in all four images. This cube was used only for demonstration purposes and was not displayed during the experiment. The right cube is an example target cube. In the four images, this cube occupies identical direction regions, but slightly different directions.

RESULTS

Our hypothesis was that direction judgment error in the interpretation of perspec-

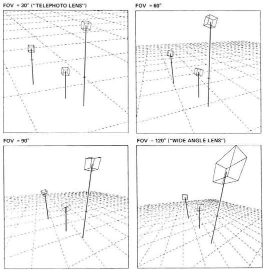

Figure 4. *Perspective images of similar cube arrangements which differ only in field of view. The third cube (on the left) is the training cube.*

tive displays varies as a function of the geometric parameters of perspective. We measured direction judgments in two orthogonal components, elevation and azimuth. It appeared reasonable that the average elevation or azimuth judgment error within each perspective condition could be used to rate the perspective conditions. This would allow selection of the best perspective parameters for communicating direction information with perspective displays. Athough the results indicate a small but significant difference in performance among the perspective conditions that is independent of target direction, they reveal much more pronounced directional effects. These directional effects have important implications for the design of perspective displays.

Statistically significant differences between perspective conditions were found for direction judgment errors, which are averaged across all target directions (see Table 1). However, the average magnitudes of the judgment errors are on the order of a few degrees and therefore do not seem to be of practical importance in display design. The main effect of field of view is statistically significant for elevation errors. However, the mean elevation judgment error is near zero for the 30-deg field of view condition and monotonically increases to only 4 deg of error at 120 deg field of view. The main effect of grid scaling is significant for elevation error, but the largest difference among the elevation error means is no more than one degree. The interaction of field of view and grid scaling is significant for both elevation and azimuth errors. There is no evident pattern to the elevation errors. The pattern of azimuth errors indicates that a more dense grid may reduce error with the extreme fields of view, 30 and 120 deg.

Directional Effects

In order to select the perspective conditions that introduce the least direction judgment error, it is necessary to consider the directional effects; that is, how direction judgments vary as a function of target direction and how perspective geometry modulates these judgments. In general, we found a consistent overestimation of target elevation, the magnitude of which is influenced by both the perspective conditions and the target azimuth. More importantly, we found a sinusoidal relationship between the azimuth direction of the target and the azimuth judgment error, in which the magnitude and direction of error varies systematically with the field of view. This sinusoidal relationship is analyzed in detail in the discussion section.

The analysis of variance for elevation judgment errors is summarized in Table 2. The main effect of target elevation is significant, as are the two-way interactions between target elevation and target azimuth, field of view, and grid scaling, respectively. Elevation judgments, averaged across azimuth direction and perspective, tend to be overestimated. When the target is in the 30-deg elevation region, the elevation is overestimated by an average of 9 deg, whereas those in the −30-deg region are overestimated by an average of 6 deg. The overall result is an expansion of 15 deg about the zero elevation plane. The overestimations are reduced at extreme elevations. The expansion between the ±30-deg elevation regions is reduced with the least dense grid (Figure 5). It is also significantly reduced as field of view increases (Figure 6). The expansion is least for a 120-deg field of view for all elevations, except those in the −60-deg elevation region, where a 90-deg field of view induces the least eleva-

Table 1

Nondirectional Sources of Judgment Error Variance

	Azimuth Error			Elevation Error		
Source of Variance	*df*	*F*	*p*	*df*	*F*	*p*
Geometric field of view	3,21	1.28		3,21	28.32	<0.0005
Grid scaling	3,21	0.60		3,21	3.22	<0.05
Field of View × Grid Scaling	9,63	2.25	<0.05	9,63	3.64	<0.001

Table 2

Directional Sources of Elevation Judgment Error Variance

Source of Variance	df	F	p
Target elevation	4,28	35.73	<0.0005
Target Elevation × Field of View	12,84	28.11	<0.0005
Target Elevation × Grid Scaling	12,84	6.74	<0.0005
Target Elevation × Target Azimuth	28,196	3.79	<0.0005

tion error. Finally, it is not surprising that elevation judgment error increases for target azimuths along the line of sight and decreases for those perpendicular to it.

The analysis of variance for azimuth judgment errors is summarized in Table 3. The main effect of target azimuth is significant, as are the two-way interactions between target azimuth and target elevation, field of view, and grid scaling, respectively. The key finding is that azimuth error varies sinusoidally with target azimuth direction, and the sinusoidal function varies systematically with the field-of-view angle. Azimuth judgment errors are analyzed in detail in the discussion section.

DISCUSSION

Direction judgment performance in computer-generated perspective images has been shown to vary significantly as a function of the perspective geometry used to create those images. The geometric field-of-view parameter significantly influences elevation and azimuth judgments, and the magnitude of the influence is a function of direction.

There is a consistent overestimation of elevation that is symmetrical about the reference altitude plane. Due to the symmetry, the average error across elevations is small, so this important effect is obscured by averaging across directions. This overestimation

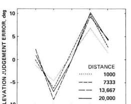

Figure 5. *Interaction of grid scaling and the stimulus elevation. Grid scaling is a function of the distance between grid and station point (see text).*

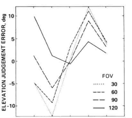

Figure 6. *Interaction of field of view and stimulus elevation.*

Table 3

Directional Sources of Azimuth Judgment Error Variance

Source of Variance	df	F	p
Target azimuth	7,49	3.15	<0.01
Target azimuth × Field of view	21,147	10.25	<0.0005
Target azimuth × Grid scaling	21,147	2.73	<0.0005
Target azimuth × Target elevation	28,196	5.89	<0.0005

is greatest for the small target elevations and decreases somewhat at the more extreme elevations. It can be reduced considerably by using a field of view greater than 90 deg, so perspective geometry has an important influence. A design implication of the expansion is that, in the absence of altitude scaling, the altitude separation of an intruding aircraft will appear to be greater than the actual separation, and terrain clearance will appear to be greater than it is in fact. Since displays can be expected to involve magnification of the altitude dimension to improve the vertical resolution, one can expect this effect to be exacerbated.

These results clearly indicate that, in addition to careful selection of perspective parameters, use of metrical symbology is required for perspective situation displays. (For examples, see McGreevy and Ellis, in press.) While pictorial instruments are indispensable for assessing dynamic configurations of objects in space, they require metrical aids for accurate measurement of situational parameters, such as direction.

The Braid

The data have demonstrated that averaging across direction conceals a most interesting interaction between target azimuth direction and field-of-view angle. Figure 7 shows the four mean azimuth error curves—one for each field-of-view angle—plotted as a function of target azimuth direction. A positive error value indicates clockwise error, and a negative error value indicates counterclockwise error. Not only is this interaction between azimuth and field of view highly significant, but the plotted curves have a distinctly sinusoidal character. The sinusoids change shape as field of view increases, gradually inverting the amplitudes. We have come to call this relationship "the braid."

For purposes of the original hypothesis, we had considered all directions within a direction region to be equivalent (as discussed in the Experimental Design section). Within these regions, however, the actual resolution of our data was higher. Thus, we had many more than just eight azimuth directions sampled, and instead had four different azimuths within each of the eight regions, for a total of 32 azimuth directions sampled for each field

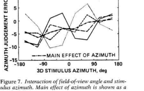

Figure 7. *Interaction of field-of-view angle and stimulus azimuth. Main effect of azimuth is shown as a dot-dash line.*

Correct Figure 7
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of view. Each of the target azimuth directions had been sampled at five different target elevations, and we took the median of the azimuth judgments across elevation as representative. Data for the eight subjects produced eight azimuth error values at each of the 32 azimuth directions, resulting in 256 points for each of the four fields of view.

By performing a least squares fit of a sixth-order polynomial to this data, we produced a generalized mean across the higher resolution azimuth data to supplement the plot of azimuth region means. Figure 8 shows data for one field of view (30 deg), and indicates the relationship between the means within each azimuth region, the median data points, and the polynomial fitted to the data points. Figure 9 shows the data points and the fitted polynomials for each of the fields of view. (Compare these data with the images in Figure 4.) The relationship between the plot of the significant interaction, based on the azimuth region means, and the four fitted polynomials can be seen by comparing Figures 7 and 10. Note the significant main effect of azimuth (in Figure 7), which is the mean of the braid.

The polynomial plots further indicate the sinusoidal nature of the relationship between azimuth error and azimuth direction. They also make even clearer the gradual transformation of the error curves as field-of-view angle changes. Observing the curves, it is evident how an average across direction could fail to produce a significant difference among fields of view: the means of the sinusoids are quite close together.

The practical interpretation of these error functions is simple to explain in terms of the stimulus images. Figure 11 shows two representative stimulus images whose fields of view produce data polynomials with opposite directions of error. The top image has a 30-deg field of view, and the bottom image has a 120-deg field of view. The grid quadrants in these images correspond to those labeled in Figure 10. Notice how well the four parts of the braid correspond to the four quadrants. The spatial interpretation of the data polynomials is that targets in particular azimuth directions are interpreted as being farther to the left or right of the heading than they are in fact, and that the bias at one extreme field of view gradually changes until it reverses at the other extreme.

The main effect of azimuth is seen as the mean of the braid, and it gives the braid a statistically significant slope. As a consequence of this slope, the magnitude and direction of error are not strictly reversed between 30- and 120-deg fields of view, though the curve *is* reversed about the mean of the braid. So, for example, by varying the field of view, targets at azimuth directions of −135 deg will be seen as about 5 deg clockwise of true direction for a 30-deg field of view, which will gradually change to about 13 deg counterclockwise for a 120-deg field of view.

Figure 9 demonstrates that, in order to

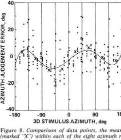

Figure 8. *Comparison of data points, the means (marked "X") within each of the eight azimuth regions, and the best fit polynomial, for a field of view of 30 deg.*

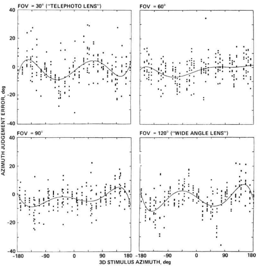

Figure 9. *Azimuth error as a function of stimulus azimuth for each field of view. A sixth-order polynomial fitted to the points summarizes the pattern of errors.*

produce an image in which target azimuth direction judgments are consistently accurate in all azimuth directions, a 60-deg field of view is the best of the four fields of view tested. That is, the amplitude of the sinusoidal azimuth error is least in the images with the 60-deg field of view. This error increases, though in opposite directions, at the narrower and wider fields of view. Thus, to effectively rate the perspectives, it is necessary to consider this sinusoidal variation of direction judgment performance.

The interaction of grid scaling and target azimuth direction is significant for azimuth error data, and, while there is no vivid pattern to the interaction, we found that with an

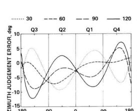

Figure 10. *Data polynomials for each field of view, a figure which we call "the braid." Quadrants are labeled as in the stimulus image diagram, Figure 2.*

enlarged scale (less dense) grid the braid is farther from the zero error line in direction quadrants Q1 and Q4, which reduces the slope of the braid. Consequently, with reduced-scale grids, the azimuth error is reduced.

Proposed Model

The distinctly braided shape of the interaction between field of view and azimuth was a surprising and provocative result, which led us to consider what aspects of the stimulus might promote such systematic errors. We noticed that the differences between the stimulus azimuth angles in the three-dimensional scene and their two-dimensional projections on the display screen varied with azimuth direction. Plotting these differences for all azimuth directions and for each of the perspective conditions, we found a family of four difference functions—one for each field of view—which not only vary sinusoidally with azimuth, but also decrease in amplitude as field-of-view angle increases. We call these difference functions the 3D-to-2D projection effect, or the *2D effect* for short. (See Figure 12, the curves marked "2DE".) These curves, however, could not in themselves account for the braid.

The fact that the distance between the subjects' eyepoint and the station point varied as a function of field of view provides the basis for another sinusoidal influence. We hypoth-

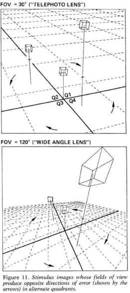

Figure 11. *Stimulus images whose fields of view produce opposite directions of error (shown by the arrows) in alternate quadrants.*

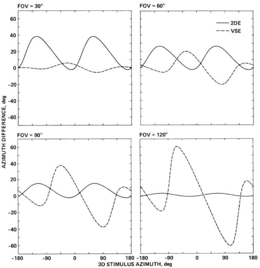

Figure 12. *Virtual space effect and 2D projection effect model curves for each field of view. Note that the effects oppose one another under these conditions and that as one increases in magnitude, the other decreases.*

esize an interpretive behavior which we call the *window assumption*, in which an observer of a pictorial display assumes that his or her eye is at the geometrically correct station point, and thus assumes that the projectors are straight. The result of this assumption is that the observer introduces systematic distortions into the perceived three-dimensional space, in an unconscious effort to maintain compatibility between it and the window assumption.

This process is quite natural, and consists merely of reprojecting points on the screen back into three-dimensional space along straight lines which all emanate from a point at the observer's eye position. When the eye is not at the geometrically correct station point, however, the true projectors are effectively bent at the point where they pierce the viewing screen. The amount of bend is a function of the distance between the station point and the actual eye position of the viewer. Another way to state this is that the amount of bend is a function of the angular difference between the field of view angle of the image and the angular subtense of the display screen as seen by the observer.

Farber and Rosinski (1978) studied a similar effect and reviewed related literature, but most previous studies have limited tasks to depth and slant judgments. Further, they assumed a significantly different virtual space effect, which differs in the determination of where each reprojected ray ends. Their assumption, if applied to the relative locations of points in three dimensions and not limited to the scaling of isolated points, can be shown to both translate and scale displayed objects along the depth dimension. Our assumption about the endpoints of reprojected rays scales objects along the axes *perpendicular* to the depth axis, by a factor that varies as a function of depth and angular subtense of the display screen. Thus, our scaling is an inverse perspective scaling transformation that straightens projectors but does not translate their endpoints along the depth dimension.

By making the window assumption, the observer hypothesizes a virtual space that would be required to exist in order to produce a given image with straight projectors. Using this as a basis for calculations, the virtual three-dimensional angle which corresponds to each three-dimensional stimulus angle may be determined. Plotting the difference between these two angles at each azimuth direction produces a family of sinusoids—one curve for each field of view—where the amplitude of the differences increases as field of view increases. We call these difference functions the *virtual space effect*. (See Figure 12, the curves marked "VSE".)

The two families of sinusoidal difference curves, the virtual space effect family and the 3D-to-2D projection effect family, represent expected influences that could bias the subjects' judgments. As a subject views the presented image and attempts to judge the true 3D angle, the fact that the angle is represented by a 2D projection, and that the true projectors may be bent, can be expected to influence the judged angle. Notice that the magnitude of the virtual space effect *increases* as field of view increases, whereas that of the 3D-to-2D projection effect *decreases* as field of view increases. Thus, it is reasonable to propose that the virtual space effect dominates judgments at wide fields of view, and that the projection effect dominates at narrow fields of view. Indeed, the virtual space effect curves are in phase with the data polynomials for the wide fields of view, and the projection effect curves are in phase with the data polynomials for the narrow fields of view. One would intuitively expect the virtual space effect to dominate at wide fields of view, as proposed, since the stimulus images subtended a narrow visual angle (18 deg) and, under these conditions of the experiment, projectors are effectively bent the most, since the station point is farthest from the subject's eyepoint.

We were curious to see if the combined effects could generate a braided family of curves similar to that seen in the interaction between field of view and azimuth direction for azimuth error data. The combination of these two effects can be obtained in a variety of ways. A simple way is to weight each one and add curves of corresponding field of view. For example, the 30-deg curve from the virtual space effect family and the 30-deg curve from the 3D-to-2D projection effect family are each weighted, and the sum plus a constant represents the combined effect. The process is repeated for the other three pairs of curves.

The weights and additive constant are determined by regression to each set of data points (four sets, one for each field of view). This produces a distinct braid that is similar to the data braid. A better fit is achieved when the component curves are shifted 22 deg counterclockwise (a shift to the left in the plots) prior to being fitted to the data. This shift has not been performed in the plots of the model components (Figure 12), but was done to produce the model braid in Figure 13. The shift could correspond to a process in which subjects make judgments relative to a line directly into the displayed space (22 deg azimuth), and then rotate the judgment 22 deg to account for the fact that the heading (zero degrees azimuth) is 22 deg counterclockwise of their actual reference direction.

We found a correspondence between the data braid (Figure 10) and the model braid (Figure 13), which suggests that the proposed influences may indeed be responsible for the data braid. However, only future experiments will show whether this proposed model is robust. A particularly interesting aspect of the model braid is that it reproduces a trend seen in the data braid that was not explicitly incorporated into the model itself. This trend is the main effect of azimuth, the overall slope of the braid. This gives us further encouragement that our model is a reasonable explanation for the pattern of azimuth errors.

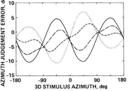

Figure 13. The combined virtual space effect and 2D projection effect, which model azimuth error as a function of stimulus azimuth for each field of view. This model "braid" compares well with the data "braid" (see Figure 10).

SUMMARY

Although perspective displays integrate spatial information and show great promise, their potential will only be realized when the design parameters of effective spatial information transfer are well understood and exploited. Human performance in deriving essential components of situational awareness, such as direction, from pictorial displays must be measured as a function of such display design parameters as perspective geometry. In our experiment, we have examined design issues that apply to panel-mounted perspective displays that include a planar grid in their symbology, subtend a narrow visual angle (e.g., approximately 18 degrees), and are mounted so they can be viewed faceon rather than obliquely.

Our analysis of direction judgments in perspective displays shows that the perspective geometry of the stimulus image has a very significant effect on direction judgment accuracy. Target elevation direction is generally overestimated, and this overestimation is made worse by use of narrow fields of view. This effect can promote overestimation of vertical separations, and it will be worsened by increased scaling of the altitude dimension.

Azimuth error varies sinusoidally with target azimuth direction and is modulated by field-of-view angle. This interaction produces a distinctly braided set of functions, which indicate that the direction of error reverses in all four direction quadrants as field of view is varied from narrow to wide angles. Of the four fields of view tested, a 60-deg field of view will produce the least overall azimuth judgment error.

Although the overestimation of elevation direction is reduced at fields of view of 90 deg and wider, it is probably better to use a 60-deg field of view for the sake of consistently accurate azimuth judgments and to use metrical symbology to assist elevation judgments. In a particular application, the required use of the displayed information will determine whether azimuth will be favored over elevation or some compromise is made, and this will guide the selection of the perspective parameters.

Geometric modeling of suspected interpretive behaviors suggests that the braided azimuth error functions are the result of biases that are induced by the difference between the 3D stimulus and its 2D projection, and by the difference between the station point and the observer's actual eye position.

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